

The collapse distance of femtosecond pulses in air

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Abstract

The conventional semi-empirical formula for collapse distance [Phys. Rev. 179, 862 (1969); Prog. Quant. Electr. 4, 35 (1975)] has been widely used in many applications. However, it is not applicable when the dispersion length is smaller than or has similar order-of-magnitude as the collapse distance. For the “enough short” pulses, there exists a threshold for the initial peak power, with which the collapse distance has a maximum value due to the competition between the Kerr self-focusing and the group velocity dispersion. New semi-empirical formulas are obtained for the collapse distance of the pulse with the initial power being less or larger than the threshold, and they can match the numerical simulations gracefully.

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I. INTRODUCTION

The ultra-short high intensity laser pulses have many applications such as laser-guided electric discharge [1], terahertz generation [2] and remote sensing [3]. Such applications need extremely high light powers which can be obtained via the collapse of pulses [4–7].

For a Gaussian beam, if the initial peak power exceeds the critical power for self-focusing, the beam will undergo collapse until the higher-order processes such as plasma or high-order Kerr effects halt the collapse [8–10]. In most cases, the collapse distance L_c (the propagation length of the self-focusing beam until collapse) can be well approximated by a semi-empirical formula [8, 10–15]

$$L_c \approx L_c^{semi} = \frac{0.367\pi n_0 r_0^2 / \lambda_0}{\sqrt{[(P_{in}/P_{cr})^{1/2} - 0.852]^2 - 0.0219}} , \quad (1)$$

where n_0 is the refractive index, r_0 is the beam width which is at $1/e^2$ level of intensity, λ_0 is the laser wavelength in vacuum, P_{in} is the initial pulse’s peak power, and P_{cr} is the critical power for self-focusing which can be written as $P_{cr} = 3.77\lambda_0^2/8\pi n_0 n_2$ with n_2 being the Kerr index. However, for the very high powers such as $P_{in} = 100P_{cr}$, experiments and numerical simulations show that the collapse distance can not be described by L_c^{semi} and a transition from a $1/\sqrt{P_{in}}$ to a $1/P_{in}$ scaling was observed [16]. Recently, simulations show that the group-velocity-dispersion (GVD) has a great influence on L_c when the pressure is relatively high, e.g., 10 atm [17].

In this paper, we investigate the collapse distance of femtosecond laser pulses in air for different temporal durations. We find that for the “very short” pulses, there exists a threshold for the initial power, with which the collapse distance has a maximum value. More importantly, new semi-empirical formulas are obtained for the collapse distance of the pulse with the initial power being less than or larger than the threshold. The new formulas take into account of the influence of GVD and can match the numerical simulations perfectly, including the cases of high pressures at which the GVD becomes more important.

II. THEORETICAL MODEL

The physical processes which halt beam collapse is still controversial. Some groups think the plasma prevents collapse (Kerr-plasma model) [18–24], while other groups believe the collapse is stopped by the high-order Kerr effects (HOKE model) [25–31]. Fortunately,

the collapse distances are almost the same for Kerr-plasma model and HOKE model [25, 32], since the HOKE and the plasma effect becomes important only after the pulse has collapsed [33], and this phenomena has also been confirmed in our numerical simulations (not shown here). In this work we study the collapse distance of the pulse for different temporal durations via HOKE model, which can be described by an extended non-linear Schrödinger equation (NLSE) [25, 27]

$$\begin{aligned} \frac{\partial A}{\partial z} = & \frac{i}{2k_0} \Delta_{\perp} A - \frac{ik''}{2} \frac{\partial^2 A}{\partial \tau^2} + \frac{ik_0}{n_0} \left(\sum_{j=1}^4 n_{2*j} |A|^{2*j} \right) A - \\ & \frac{ik_0}{2} \frac{\omega_{pe}^2}{\omega_0^2} A - \frac{A}{2} \sum_{l=O_2, N_2} \left(\frac{W_l(I) U_l}{|A|^2} (\rho_{at,l} - \rho_{e,l}) \right), \end{aligned} \quad (2)$$

where A represents the envelope of the electric field, and z denotes the propagation distance. $k_0 = 2\pi/\lambda_0$ ($\lambda_0 = 800$ nm) is the central wave number. The Laplacian operator $\Delta_{\perp} = \partial_r^2 + 1/r \partial_r$ denotes the beam transverse diffraction. The remaining terms in the right-hand-side of Eq.(1) account for the group velocity dispersion with the second order dispersion coefficient $k'' = 0.2$ fs²/cm, Kerr and high-order Kerr effect with nonlinear refractive index $n_2 = 1.2 \times 10^{-19}$ cm²/W, $n_4 = -1.5 \times 10^{-33}$ cm⁴/W², $n_6 = 2.1 \times 10^{-46}$ cm⁶/W⁴, and $n_8 = -0.8 \times 10^{-59}$ cm⁸/W⁴ [34, 35], plasma defocusing with the plasma oscillation frequency $\omega_{pe} = \sqrt{q_e^2 \rho / m_e \epsilon_0}$ (q_e is the electron charge, m_e is the electron mass and ρ is the free electron density.), the energy loss caused by MPA. $W_{N_2}(I)$ and $W_{O_2}(I)$ are the photoionization rate of N₂ and O₂. $\rho_{at, N_2} = 2.1 \times 10^{25}$ m⁻³ and $\rho_{at, O_2} = 5.7 \times 10^{24}$ m⁻³ are the density of N₂ and O₂ molecular at 1 atm. ρ_{e, N_2} and ρ_{e, O_2} are free electron density ionized by N₂ and O₂ ($\rho = \rho_{e, N_2} + \rho_{e, O_2}$) which can be calculated by the following equations

$$\frac{\partial \rho_{e, N_2}}{\partial \tau} = W_{N_2}(I) (\rho_{at, N_2} - \rho_{e, N_2}) , \quad (3)$$

$$\frac{\partial \rho_{e, O_2}}{\partial \tau} = W_{O_2}(I) (\rho_{at, O_2} - \rho_{e, O_2}) . \quad (4)$$

The photoionization rate of N₂ and O₂ is obtained by the Keldysh-PPT (Perelomov-Popov-Terent'ev) formula [36].

The initial pulse investigated in this paper is Gaussian beam which can be written as:

$$E(r, \tau, 0) = \sqrt{\frac{2P_{in}}{\pi r_0^2}} \exp \left(-\frac{r^2}{r_0^2} - \frac{\tau^2}{\tau_0^2} \right) , \quad (5)$$

where τ_0 is the temporal duration.

III. RESULTS AND DISCUSSIONS

Follow Li et al. [17], we define the collapse distance as the distance between the light source and the position where the laser beam has the smallest radius. Fig. 1 shows the evolution of the beam radius of pulses which have different temporal durations (80 fs and 300 fs). It can be seen that the collapse distances of the two pulses have big difference (207 m and 135 m). In contrast, the semi-empirical formula in Eq. (1) gives $L_c^{semi} = 129$ m since it does not take into account of the pulse's duration. Therefore, the temporal duration of pulse may have a large influence on the collapse distance in some circumstances.

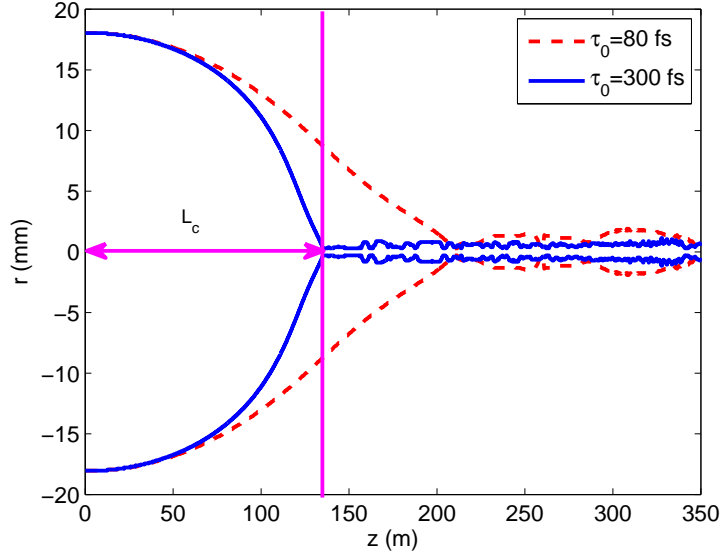


FIG. 1. Beam radius of the pulse at different propagation distances. The input beam are Gaussian pulses with $r_0 = 18$ mm and $P_{in}/P_{cr} = 20$.

From now on, we focus on the influence of the temporal duration to the collapse distance. In the simulations, we consider the Gaussian pulse with two different beam radius ($r_0 = 2$ mm and $r_0 = 9$ mm).

Fig. 2 shows the variation of the collapse distances with different initial powers and temporal durations. Here $r_0 = 2$ mm, P_{in}/P_{cr} varies from 1.5 to 6, and τ_0 varies from 10 to 100 fs. For comparisons, the collapse distances calculated by L_c^{semi} is also given and shown in solid line. We can observe that L_c^{semi} can approximate L_c very well for the long durations such as $\tau_0 = 60$ fs, 80 fs and 100 fs, but it fails for the short ones, e.g., $\tau_0 = 10$ fs, 15 fs and 30 fs. In order to see more clearly how the collapse distances of the short-temporal-duration

pulses change with the input power, we show the results in Fig. 3 for $\tau_0=10$ fs, 15 fs, 30 fs and P_{in}/P_{cr} being in the range between 1.5 and 12. It can be seen from this figure that the collapse distances of pulses with these short durations have a maxima and tend to L_c^{semi} in the limit of P_{in}/P_{cr} going to infinity. We define the input peak power which leads to the maximum collapse distance as P_{MC} . If $P_{in} < P_{MC}$, the collapse distance increases with P_{in} , and if $P_{in} > P_{MC}$, the collapse distance decreases with P_{in} .

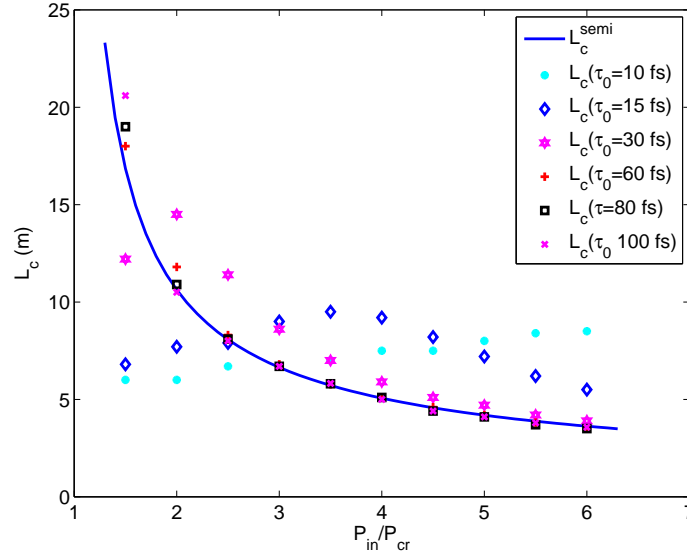


FIG. 2. Variation of the collapse distance of pulses with the initial peak power. The input beam are Gaussian pulses with $r_0 = 2$ mm.

Fig. 4 shows the collapse distances for the pulses with $r_0=9$ mm. P_{in}/P_{cr} varies from 2 to 30, and $\tau_0 = 40$ fs, 80 fs, 200 fs, 450 fs, respectively. From Fig. 4 we can get the same conclusion as shown in Fig. 2 and 3 that L_c has a maximal value when the temporal duration is very short. It is worth pointing out that “very short” is relative to the radius of pulses. For example, $\tau_0=80$ fs is “very short” when $r_0=9$ mm, whereas it is not “very short” when $r_0=2$ mm.

The phenomena that L_c^{semi} can not fit L_c well for the “very short” pulses is due to GVD. The normal GVD enables power exchanges between different time slices of the pulse and disperses the latter in time, and this may contribute to maintaining the pulse self-guiding at smaller intensity levels [37–39]. If the pulse’s temporal duration is long enough, the effects of GVD can be neglected when the propagation distance is less than the collapse

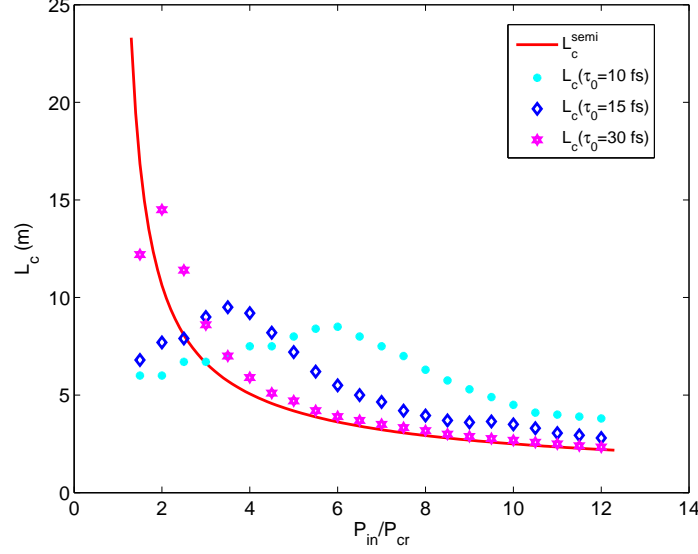


FIG. 3. Variation of the collapse distance of the short pulses with the initial peak power. The input beam are Gaussian pulses with $r_0 = 2$ mm.

distance, and thus L_c^{semi} can fit the collapse distance very well. However, if the pulse's temporal duration is very short, the GVD effect will have a strong competition to the Kerr self-focusing effect when the initial power is small. Let's consider an extremely short pulse with small initial peak power. In this extreme situation, the GVD effect overwhelms the Kerr self-focusing, and the collapse distance equals the propagation distance at which the peak intensity decreases to P_{cr} . At the same time, the temporal duration varies with z as $\tau_1(z) = \tau_0[1 + (z/2L_{GVD})^2]^{1/2}$ [40], here $L_{GVD} = \tau_0^2/2k''$ denotes the dispersion length [12], and thus the peak power can be written as $P(z) = P_{in}/\tau_1(z)$. The collapse distance in this case can be approximated by

$$L_c^{extreme} = 2L_{GVD}\sqrt{(P_{in}/P_{cr})^2 - 1}. \quad (6)$$

This equation may account for the phenomena that the collapse distances of short pulses increase with P_{in}/P_{cr} when $P_{in} < P_{MC}$ (Figs. 2-4), though Eq. 6 can not be used to estimate L_c accurately in general cases.

With closer inspections for Figs. 2-4, we find that L_c is proportional to P_{in}/P_{cr} when $P_{in} < P_{MC}$, and the slope is dependent on the temporal duration and the radius of pulse. Taking into account the GVD effects, we finally obtain the new semi-empirical formulas as

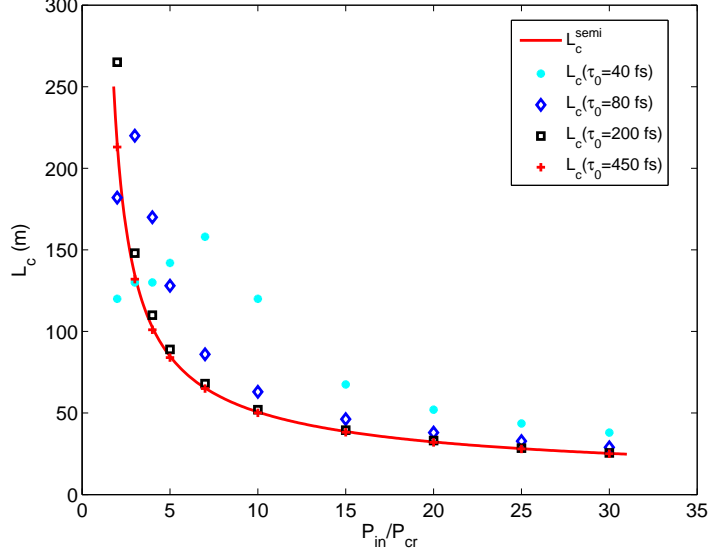


FIG. 4. Variation of the collapse distance with the initial peak power (P_{in}/P_{cr} range from 2 to 30). The input beam are Gaussian pulses with $r_0 = 9$ mm. The temporal durations of the pulses are 40 fs, 80 fs, 200 fs, 450 fs.

follow

$$L_c^{semi,new} = \begin{cases} 0.225L_{GVD}\frac{P_{in}}{P_{cr}} + 0.320L_{DF}, & \text{if } P_{in} < P_{MC}, \\ L_c^{semi} \times 1.878^N, & \text{otherwise,} \end{cases} \quad (7a)$$

$$(7b)$$

where $L_{DF} = \pi n_0 r_0^2 / \lambda_0$ is the Rayleigh length, which account for the contribution from the diffraction. N is a dimensionless parameter and

$$N = \frac{L_c^{semi}}{L_{GVD}}, \quad (8)$$

which characterizes the relative importance of the Kerr self-focusing and the GVD. When $N \gg 1$ GVD dominates, while for $N \ll 1$ Kerr self-focusing dominates.

From Eq. (7), we can see that when $P_{in} < P_{MC}$ the collapse distance is mainly determined by the GVD and the diffraction, otherwise, all the GVD, the diffraction and the Kerr self-focusing plays important roles. The input power threshold P_{MC} can be obtained via equaling Eq. (7a) and Eq. (7b). The relations between P_{MC} , the duration τ_0 and the radius r_0 of the laser pulse are shown in Fig. 5. It can be seen that P_{MC} increases with r_0 . At the same time, P_{MC} decreases with τ_0 , and it reduces to P_{cr} in the the limit of $\tau_0 \rightarrow \infty$.

In real applications of the new semi-empirical formula, we do not need the value of P_{MC} .

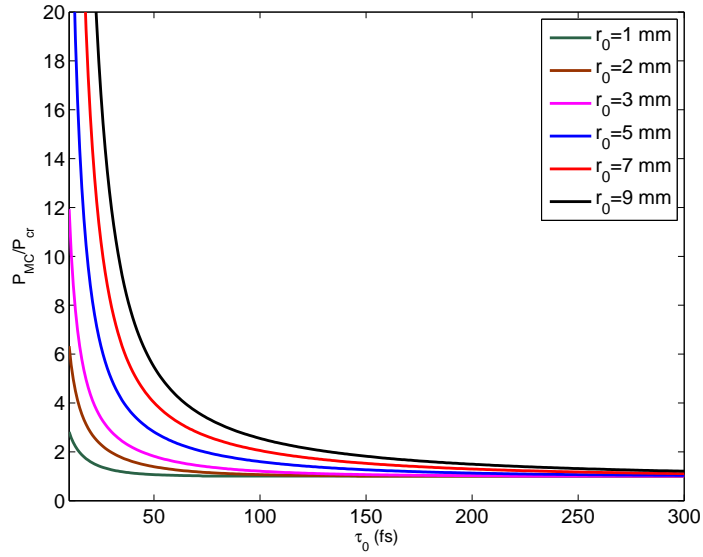


FIG. 5. Dependence of P_{MC} on the pulse's duration and pulse's radius.

Notice that Eq. (7a) is a monotonically increasing function of P_{in} , while Eq. (7b) is a monotonically decreasing function of P_{in} , and the value of input power corresponding to the intersection of these two functions is P_{MC} . Therefore, we can directly use $\min\{\text{Eq. (7a), Eq. (7b)}\}$ to obtain the collapse distance L_c for a given P_{in} , without knowing the value of P_{MC} .

Fig. 6 shows the comparisons of the collapse distances between the new semi-empirical formula, the direct numerical simulations, and the conventional semi-empirical formula. It can be seen that the new semi-empirical formula agree with the simulation results very well, for both the long and short pulses.

Now we turn to the case shown in Fig. 1, in which the collapse distances are obtained by numerical simulations. Basing on the new semi-empirical formula given in Eq. (7), we obtain the collapse distance of 214 m (207 m in simulation) for 80 fs and 134 m (135 m in simulation) for 300 fs.

Eq. (7) can also be used to calculate the collapse distance at high pressures. L_{GVD} is inversely proportional to the pressure, and play more important roles at high pressure. For example, in the work of [17], the authors obtained the collapse distance of the pulse with $\tau_0 = 50$ fs and $r_0 = 1.2$ mm, $P_{in} = 4P_{cr}$ is 2.3 m at 10 atm, while the new semi-empirical formula gives $L_c = 2.2$ m, perfectly matched.

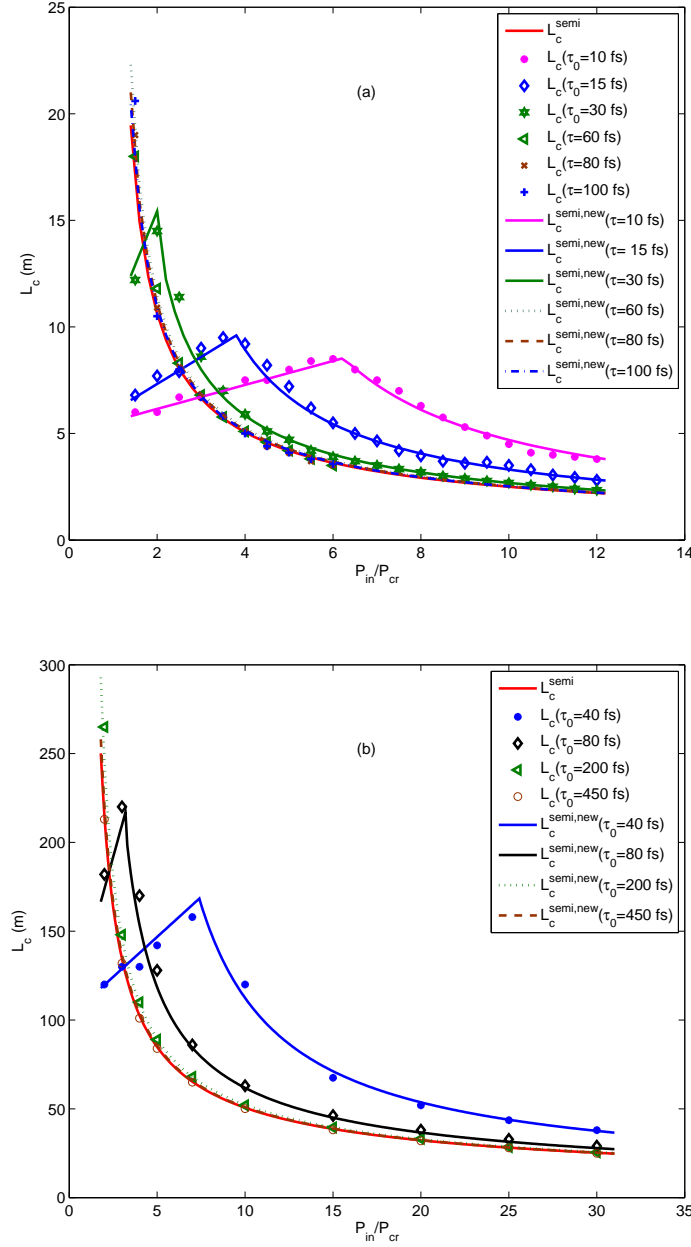


FIG. 6. The collapse distance predicted by the new semi-empirical formula. The input beam are Gaussian pulses with (a) $r_0 = 2$ mm, (b) $r_0 = 9$ mm.

IV. CONCLUSION

GVD plays important roles in the evolution of the laser pulses in air. For the short pulses, due to the competition between GVD and Kerr self-focusing, there exists a threshold P_{MC} for the initial power, with which the collapse distance has a maximum value. If the

initial power is less than the threshold, the collapse distance is proportional to the input power. If the initial power is larger than the threshold, the collapse distance decreases with the the input power, and is larger than that obtained by the conventional semi-empirical formula. Taking into account the effects of GVD, we present new semi-empirical formulas, which match the numerical simulations very well. The formulas can also be applicable to the cases for high pressures in which the GVD effects become more important.

V. ACKNOWLEDGMENTS

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